6.3 Graph a Line Using Intercepts

Joanne loves reading. She has \$48 to spend at her favourite used book store. She likes comic books, which cost \$4 each, and novels, which cost \$6 each. What combinations of comic books and novels can Joanne buy?

Tools
- grid paper

Investigate

How can you understand the meaning of intercepts of a linear graph?

Refer to the information above.

1. If Joanne buys only comic books, and no novels, how many can she buy?

2. If Joanne buys only novels, and no comic books, how many can she buy?

3. Let x be the number of comic books. Let y be the number of novels. Write each combination in steps 1 and 2 as an ordered pair (x, y).

4. Plot the ordered pairs from step 3 on a graph. Join the points with a straight line.

5. **a)** If Joanne buys a combination of comic books and used novels, what combinations can she buy?

b) Explain how you found these combinations.

6. **Reflect** Look at your graph.

a) Explain how you can use the graph to discover combinations that work.

b) You must be careful when using a linear model. In this situation, the point $\left(\frac{3}{2}, 7\right)$ has no meaning, even though it is on the line. Why not? *Hint*: What does x represent?

c) Identify two other points that are on the line, but have no meaning. Explain why they have no meaning.

Lines can be written in many forms.

$y = -\frac{2}{3}x + 8$ **Slope *y*-intercept form.**

$2x + 3y - 24 = 0$ **Standard form.**

Both of these equations describe the same line. You could also express this line in another way: $2x + 3y = 24$.

Although this form has no special name, it is useful for graphing purposes.

Example 1 Calculate Intercepts

The following equation can be used to model the situation described in the Investigate:

$$4x \quad + \quad 6y \quad = \quad 48$$

\$4 × (number of comic books) + \$6 × (number of novels) = Total spent

a) Determine the *x*- and *y*-intercepts of the equation $4x + 6y = 48$.

b) Use the intercepts to graph the line.

Solution

a) Find the *x*-intercept.

At the *x*-intercept, the value of *y* is 0.

$$4x + 6(0) = 48$$
$$4x = 48 \qquad \text{Solve for } x.$$
$$\frac{4x}{4} = \frac{48}{4}$$
$$x = 12$$

The *x*-intercept is 12.
The point (12, 0) is on the line.

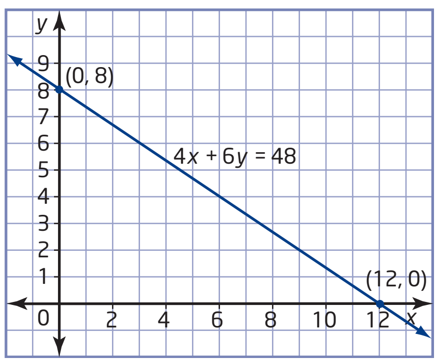
Find the *y*-intercept.

At the *y*-intercept, the value of *x* is 0.

$$4(0) + 6y = 48$$
$$6y = 48 \qquad \text{Solve for } y.$$
$$\frac{6y}{6} = \frac{48}{6}$$
$$y = 8$$

The *y*-intercept is 8.
The point (0, 8) is on the line.

b) Plot the intercepts to graph this relation.



You can use this graph to find other points that satisfy the equation, such as (3, 6), (6, 4), and (9, 2).

Reflect

Be careful when using a linear model. In this example, the point $\left(11, \frac{2}{3}\right)$ is on the line, but has no meaning. Why not? *Hint*: What does *y* represent?

Example 2 Use Intercepts to Graph a Line

For each linear relation, determine the x- and y-intercepts and graph the line.

a) $2x - y = 7$ **b)** $3x - 5y + 15 = 0$

Solution

a) Find the x-intercept.
Substitute $y = 0$.

$$2x - 0 = 7$$
$$2x = 7 \quad \text{Solve for } x.$$
$$\frac{2x}{2} = \frac{7}{2}$$
$$x = \frac{7}{2}$$

The x-intercept is $\frac{7}{2}$ or 3.5.

The point (3.5, 0) is on the line.

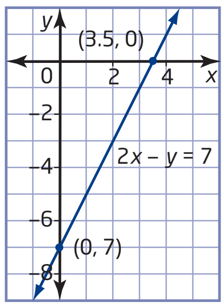
Find the y-intercept.
Substitute $x = 0$.

$$2(0) - y = 7$$
$$-y = 7 \quad \text{Solve for } y.$$
$$\frac{-y}{-1} = \frac{7}{-1}$$
$$y = -7$$

The y-intercept is -7.
The point $(0, -7)$ is on the line.

I could do this mentally. I just cover up the x-term and solve $-y = 7$ in my head.

Plot the intercepts.
Draw a line through the intercepts.
Label the line with the equation.



b) $3x - 5y + 15 = 0$

Find the x-intercept.
Substitute $y = 0$.

$$3x - 5(0) + 15 = 0$$
$$3x + 15 = 0$$
$$3x = -15$$
$$x = -5$$

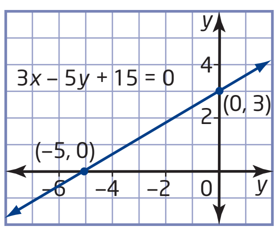
The x-intercept is -5.
The point $(-5, 0)$ is on the line.

Find the y-intercept.
Substitute $x = 0$.

$$3(0) - 5y + 15 = 0$$
$$-5y + 15 = 0$$
$$-5y = -15$$
$$y = 3$$

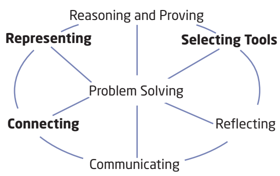
The y-intercept is 3.
The point (0, 3) is on the line.

Plot the intercepts.
Draw a line through the intercepts.
Label the line with the equation.



Example 3 Find the Slope Using the Intercepts

Determine the slope of the line whose x-intercept is -4 and y-intercept is -6.



Solution

Method 1: Apply Algebraic Reasoning

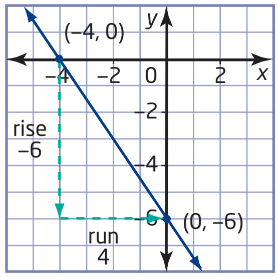
The points $(-4, 0)$ and $(0, -6)$ are on the line. Substitute these into the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-6 - 0}{0 - (-4)}$$
$$= \frac{-6}{4}$$
$$= -\frac{3}{2}$$

The slope of the line is $-\frac{3}{2}$.

Method 2: Apply Geometric Reasoning

Graph the line by plotting the intercepts. Read the rise and the run from the graph.

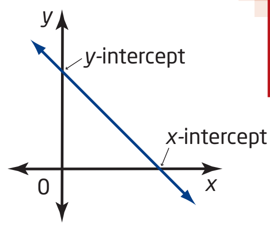


I go down 6 units to move from the first point, $(-4, 0)$, to the second point, $(0, -6)$, so the rise is -6. The run is 4.

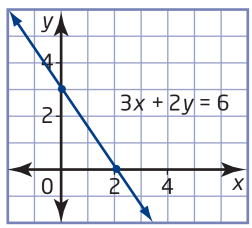
$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-6}{4}$$
$$= -\frac{3}{2}$$

Key Concepts

- The x-intercept is the x-coordinate of the point where a line crosses the x-axis. At this point, $y = 0$.
- The y-intercept is the y-coordinate of the point where a line crosses the y-axis. At this point, $x = 0$.



- For some equations, it is easy to graph a line using intercepts. For example, for $3x + 2y = 6$:
  - When $x = 0$, $y = 3$.
  - When $y = 0$, $x = 2$
  - Plot the two intercepts and draw a line through them.



Communicate Your Understanding

C1 A line has an x-intercept of 3 and a y-intercept of -4. Use the intercepts to state the coordinates of two points on this line.

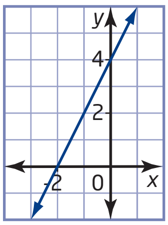
C2 **a)** Is it possible for a line to have no y-intercept? Explain.

b) Give an example of a line that has no y-intercept. Write the equation and sketch its graph.

C3 A line has a y-intercept of -2, but has no x-intercept. Describe this line in words, and sketch its graph.

Practise

1. Identify the x- and y-intercepts of each graph, if they exist.

a)


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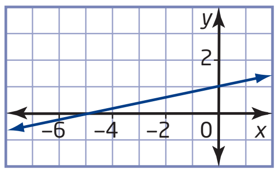
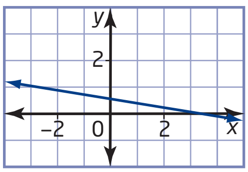
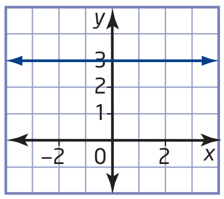


c)


d)


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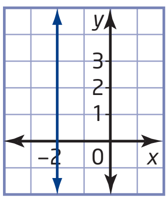


For help with questions 2 and 3, see Example 1.

2. For each part, plot the intercepts and graph the line.

	x-intercept	y-intercept
a)	2	5
b)	–3	3
c)	1.5	–4
d)	none	6
e)	4	none

3. Determine the x- and y-intercepts and use them to graph each line.

a) $2x + 3y = 12$

b) $3x + y = 6$

c) $x - 4y = 4$

d) $-5x + 2y = 10$

e) $4x = 12$

f) $3y = -9$

g) $4x + 2y = 6$

h) $x - 3y = 5$

Connect and Apply

For help with question 4, see Example 2.

4. Draw a graph and determine the slope of each line using the rise and run from the graph.

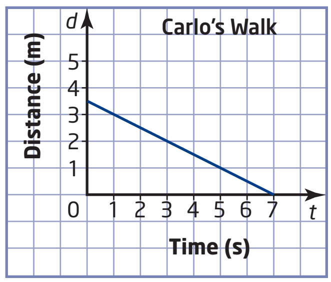
	x-intercept	y-intercept
a)	5	–5
b)	–2	3
c)	3	none
d)	2.5	–4

For help with question 5, see Example 3.

5. Find the slope of each line, given the x- and y-intercepts, using the slope formula.

	x-intercept	y-intercept
a)	6	5
b)	3	–4
c)	–6	3
d)	none	$\frac{1}{2}$

6. The distance-time graph shows Carlo's motion in front of a motion sensor.



a) Identify the d-intercept and explain what it means.

b) Identify the t-intercept and explain what it means.

c) Describe the instructions you would give to a person walking in front of a motion sensor to reproduce this graph.

7. Consider the line $x + 4y = -4$. To graph this line, you could

- determine the x- and y-intercepts
- create a table of values
- use the equation to find the coordinates of three points on the line

Which method of graphing do you prefer in this case? Explain.

8. A candle burns at a constant rate of 2.5 cm/h. The candle is 15 cm tall when it is first lit.

   a) Set up a graph of length, *l*, in centimetres, versus time, *t*, in hours, and plot the *l*-intercept.

   b) Should the slope of this linear relation be positive or negative? Explain.

   c) Graph the line.

   d) What is the length of the candle after
      - 3 h?
      - 4.5 h?

   e) Identify the *t*-intercept and explain what it means.

   f) Explain why this graph has no meaning below the *t*-axis.

9. Explain and use sketches to support your answers to each question.

   a) Is it possible for a line to have no *x*-intercept?

   b) Is it possible for a line to have more than one *x*-intercept?

   c) Is it possible for a line to have no *x*-intercept and no *y*-intercept?

10. **Use Technology** Use *The Geometer's Sketchpad*® to model and explore in more depth the problem posed in the Investigate.

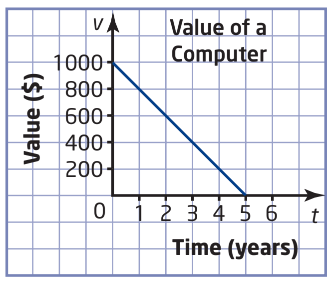
    a) Construct a geometric model for the problem.
       - Open *The Geometer's Sketchpad*® and begin a new sketch.
       - From the **Graph** menu, choose **Show Grid**.
       - Select the *x*-axis and, from the **Construct** menu, choose **Point On Axis**. Click and drag the point until it is at (12, 0).
       - Construct a point on the *y*-axis and move it to (0, 8).
       - Select the two intercept points and, from the **Construct** menu, choose **Line**.

    b) Explore the effects on the linear model when the intercepts change. What happens to the slope of the line in each situation?
       - The *x*-intercept is increased.
       - The *x*-intercept is decreased.
       - The *y*-intercept is increased.
       - The *y*-intercept is decreased.

    c) Suppose that the price of comic books goes up. What effect will this have on the linear model? What impact will this have on Joanne's buying power? Explain your reasoning.

    d) Suppose that the store has a 50% off sale on novels. Repeat part c) for this scenario.

11. When you buy a computer, its value depreciates (becomes less) over time. The graph illustrates the value of a computer from the time it was bought.



    a) How much did the computer originally cost?

    b) After what period of time does the computer no longer have any value?

    c) What is the slope of this graph and what does it mean?

Extend

12. Refer to question 11. Sometimes depreciation is calculated differently. Suppose that each year, the computer's value becomes 50% of its previous year's value.

a) Construct a table of values of the computer's value versus time for the first 5 years after the date of purchase.

b) Graph this relation. Is it linear or non-linear? Explain.

c) After how many years will the computer be worth
- less than 10% of its original value?
- zero?

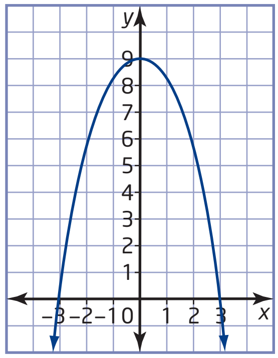
d) Does the t-intercept exist? If yes, what is it? If no, why not?

e) Compare this graph with the one in question 11. Under which system does the computer's value depreciate faster? Explain.

Making Connections

The graph in question 13 illustrates a special type of non-linear relationship called a *quadratic relation*. You will study these in depth in grade 10.

13. a) How many x-intercepts does this graph have? What are they?



b) How many y-intercepts does this graph have? What are they?

c) Sketch the graph of a relation that has two y-intercepts.

d) Sketch the graph of a relation that has three x-intercepts.

e) Sketch the graph of a relation that has two x-intercepts and two y-intercepts.

14. Math Contest The ordered pair (x, y) locates a point on a plane. The ordered triple (x, y, z) can be used to locate a point in three-dimensional space. For example, to locate the point A(2, 3, 4), start at the origin, (0, 0, 0), move 2 units right, 3 units up, and 4 units out of the page. Describe how to locate the points B(5, −3, 1) and C(−2, 0, 4). If you were to join the three points, what would the shape of the resulting figure be?

15. Math Contest Start with the equation $6x - 2y - 18 = 0$. Write this equation in the form $y = m(x - a)$. What information does the value of a give you about the graph of this line? Repeat this investigation using any other line written in standard form. Draw conclusions about the form $y = m(x - a)$.

Use *The Geometer's Sketchpad®* to Explore Parallel and Perpendicular Lines

The geometric properties of parallel and perpendicular lines make them very useful in mathematics. How can you recognize whether two equations represent parallel or perpendicular lines?

Tools

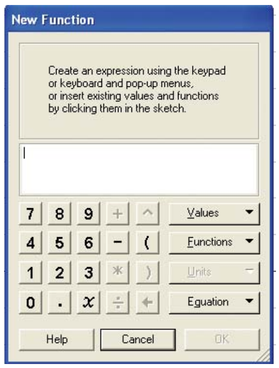
- *The Geometer's Sketchpad®*
- protractor

Technology Tip

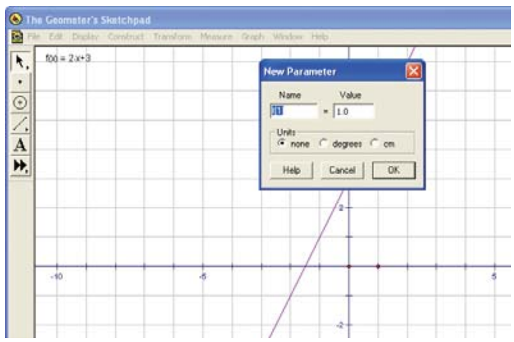
A *parameter* is a variable that is assigned a specific value. By setting the slope as a parameter, you can change its value either by hand or automatically and immediately see the effect on the line.

Investigate

How are the slopes of parallel and perpendicular lines related?



1. Graph the line $y = 2x + 3$ using *The Geometer's Sketchpad®*.
- Open *The Geometer's Sketchpad®* and begin a new sketch.
- From the **Graph** menu, choose **Show Grid**.
- From the **Graph** menu, choose **New Function**. A function calculator screen will appear.
- Click on $2 * x + 3$.
- Click on **OK**.
- From the **Graph** menu, choose **Plot Function**. The line $y = 2x + 3$ should appear.



2. Set a **parameter** for the slope of a new line.
- Deselect by clicking somewhere in the white space.
- From the **Graph** menu, choose **New Parameter**. A dialogue box with the heading **New Parameter** will appear.
- Type m in the **Name** field. Click on **OK**. Leave the **Value** set at 1.0. A parameter measure, m, will appear near the left side of the screen. You will use this in the next step.

Literacy Connections

The Geometer's Sketchpad® uses a special notation called *function notation*. In function notation, you replace y with $f(x)$.

Regular notation:
$y = 2x + 3$

Function notation:
$f(x) = 2x + 3$

To read function notation aloud, you say "f of x equals..." or "f at x equals...." You can also use other letters, such as $g(x)$ and $h(x)$. This is useful when you are working with more than one equation at a time.

3. a) Graph a line $y = mx + 2$ with a moveable slope.

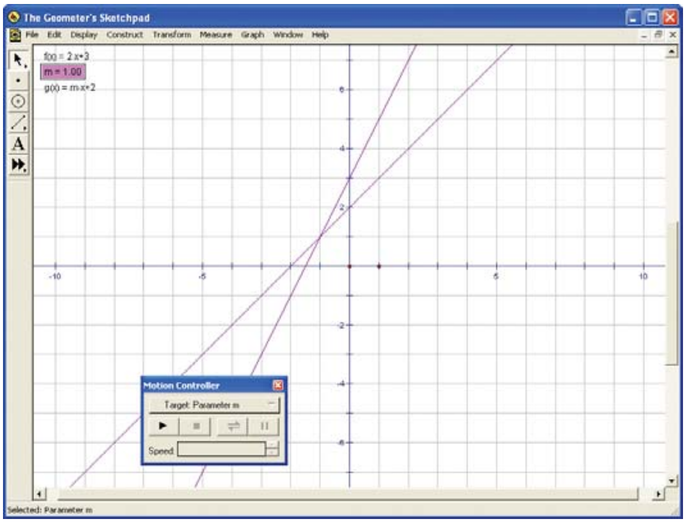
- Deselect by clicking in the white space.
- From the **Graph** menu, choose **New Function**.
- When the dialogue box appears, click on the parameter measure m.
- Click on $* \ x + 2$
- Click on **OK**.
- From the **Graph** menu, choose **Plot Function**.
  The line $y = x + 2$ will appear.

b) Why did a line with a slope of 1 appear? *Hint:* Think about how you set the parameter.

4. Change the slope of $y = mx + 2$ automatically.

- Deselect.
- Right click on the parameter measure m.
- Select **Animate Parameter**.
- Watch the line and the value of m. Describe what happens.

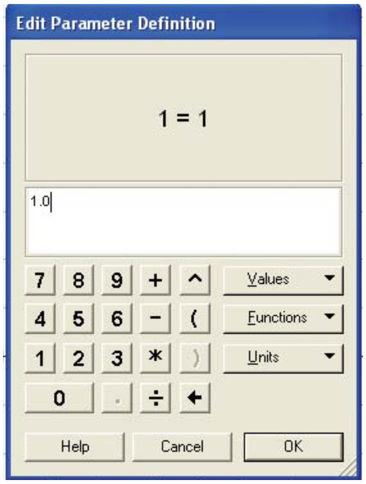
5. Explore the **Motion Controller**. When you click on **Animate Parameter**, a **Motion Controller** dialogue box appears. Experiment with the different controls. Write a brief explanation of what each command does.



6. Change the slope of $y = mx + 2$ by hand.
   - Stop the **Motion Controller** and close its window.
   - With the parameter measure m still selected, type the + sign on the keyboard several times. Describe what happens to the line and the slope. Repeat for the − sign.

7. a) Try to find the value that makes the line $y = mx + 2$ parallel to $y = 2x + 3$. To set a precise value, do the following:
   - Right click on the parameter measure m.
   - Choose **Edit Parameter**. An **Edit Parameter Definition** dialogue box will appear.
   - Type in a value. Are the lines parallel? If not, repeat the above step until they are.

   b) For what value of m is the line $y = mx + 2$ parallel to the line $y = 2x + 3$?



8. Find the value of m that makes $y = mx + 2$ perpendicular to $y = 2x + 3$. You can use a protractor to measure the angle of intersection of the two lines.

9. Find the slopes of lines that are parallel and perpendicular to each line given. Organize your results in a table like this.

Given Line	Slope of Given Line	Slope of Parallel Line	Slope of Perpendicular Line
$y = -x + 2$			
$y = \frac{2}{3}x - 4$			
line of your choice			

10. **Reflect** Look at your results.

   a) Describe how the slopes of parallel lines are related.

   b) Describe how the slopes of perpendicular lines are related. *Hint:* Explore the products of the slopes of perpendicular lines.